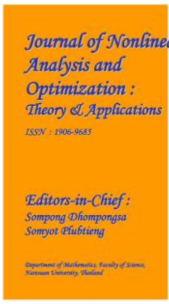
Journal of **N**onlinear **A**nalysis and **O**ptimization
Vol. 15, Issue. 2, No.3 : 2024
ISSN : **1906-9685**



AN APPLICATION OF NEUTROSOPHIC SOFT SET FOR SOLVING A MULTI-CRITERIA DECISION MAKING PROBLEM IN INVESTMENT PORTFOLIO SELECTION

P.Subashini, C.Jayabharathi, Research Scholar, Department of Mathematics, A.D.M. College for women(Autonomous), Affiliated to Bharathidasan University, Nagapattinam, Tamil Nadu, India. subaselvam104@gmail.com
R.Sophia Porchelvi, Associate Professor, Department of Mathematics, A.D.M. College for women(Autonomous), Affiliated to Bharathidasan University, Nagapattinam, Tamil Nadu, India.

ABSTRACT

In this paper, a new approach for solving a multi-criteria decision making problem that uses neutrosophic soft matrix is introduced. A solving procedure has been developed by constructing choice matrix, max-min product and comparison matrix. Furthermore, a numerical example is provided to illustrate the feasibility of the proposed method.

Keywords: Neutrosophic soft set, Neutrosophic soft matrix, Max-min product, choice matrix, comparison matrix.

1. INTRODUCTION

Decision making theory is the process of selecting a right and effective choices from two or more alternatives for the purpose of achieving desired result. Many real life problems have been solved in decision making method. Fuzzy set theory was proposed by Lotfi A. Zadeh [1] in 1965. Fuzzy set thoery has been applied to many fields of operations research. Fuzzy decision making environments provide several methods to solve multi criteria decision making problems.

Neutrosophic set (NS) was introduced by Florentin Smarandache [3] in 1998. It is a mathematical tool for handling problems involving imprecise, indeterminacy, and inconsistent data. Soft set theory was introduced by Molodtsov in 1999 [4]. Maji [5] combined the concept of soft set and neutrosophic set together by introducing a new concept called Neutrosophic soft set (NSS). Recently, Sujit Das, Samarjit kar(2017) [7] used the concept of choice matrix and Max-min product of Intuitionistic fuzzy soft set and Neutrosophic soft set [8] to solve decision making problems. Murat, Necip(2019) [9] introduced the comparison matrix for neutrosophic soft matrix and he applied the same in medical diagnosis.

This paper presents a new modified method to solve multi criteria decision making problem by integrating the methods given in [8] and [9]. The final results are obtained from the comparison matrix based on the maximum score value. This paper consists the following sections: Section 2 contains basic definitions related to neutrosophic soft set. The proposed method and its application are discussed in section 3. Conclusion appear in section 4.

2. PRELIMINARIES

Definition 2.1 (Neutrosophic Set)

Let U be an universe of discourse. The neutrosophic set A in U is expressed by $A = \{\langle x: T_{A(X)}, I_{A(X)}, F_{A(X)}\rangle, x \in U\}$, where the characteristic functions $T, I, F : U \to]-0, 1^{+}[$ respectively define the degree of membership, the degree of indeterminacy and the degree of non-membership of the element $x \in U$ to the set A with the condition, $-0 \le T_{A(X)} + I_{A(X)} + F_{A(X)} \le 3^{+}$.

Definition 2.2 (Neutrosophic Soft Set)

Let U be a universe of discourse and E be a set of parameters. Let $NS(U)$ denotes the set of all neutrosophic subsets of U and $A \subset E$. A pair $(N_{\{A\}}, E)$ is called a neutrosophic soft set over U, where $N_{\{A\}}$ is a mapping given by $N_{\{A\}}: E \to NS(U)$.

Definition 2.3 (Neutrosophic Soft Matrix: NSM)

Let $(N_{\{A\}}, E)$ be a NSS over the universe U. Let E be a set of parameters and $A \subseteq E$. Then a subset of $U \times E$ uniquely defined by the relation $\{(x, e): e \in A, x \in N_{\{A\}}(e)\}$ and denoted by M_A=$(N_{\{A\}}, E)$. The relation characterized by the truth function T_A:$U \times E \to [0,1]$, indeterminacy $I_A : U \times E \to [0,1]$, and the falsity function $F_A : U \times E \to [0,1]$. M_A is represented as M_A=$\{(T_A(x, e), I_A(x, e), F_A(x, e)) : 0 \le T_A + I_A + F_A \le 3, (x, e) \in U \times E\}$. Now if the set of universe $U = \{x_1, x_2, \ldots x_m\}$ and the set of parameters $E = (e_1, e_2, \ldots, e_n)$, then M_A can be represented by a matrix as follows,

$$M_A = (a_{ij})_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Where $a_{ij} = \left(T_A(x_i, e_j), I_A(x_i, e_j), F_A(x_i, e_j)\right)$.

Definition 2.4 (Neutrosophic Choice Matrix:NCM)

Neutrosophic choice matrix is a square matrix whose rows and columns both indicates parameters. If μ is a NCM, then its element $\mu(i, j)$ is defined as, $\mu(i, j) = (1, 0.5, 0)$ when i^{th} and j^{th} both parameters are the choice parameters of decision maker. $\mu(i, j) = (0, 0.5, 1)$ when at least one of the i^{th} or j^{th} parameters be not under choice of the decision maker.

In combined NCM, denoted by μ^c, rows indicate choice parameters of single decision maker and columns indicate combined choice parameters of the remaining decision maker.

Definition 2.5 (Max-min product of NSM)

Two NSMs, $(N,A)_{m\times n} = [a_{ij}]_{m\times n}$ and $(N,B)_{m\times n} = [b_{ij}]_{m\times n}$ are said to conformable for product, if the number of columns of (N,A) is equal to the number of rows of (N,B). The product of NSM is defined by $(N,P) = (N,A) \otimes (N,B) = (c_{ik})_{m\times p}$, where, $c_{ik} = \left(\max_{j=1} n\ min\ \{T_{a_{ij}}, T_{b_{jk}}\}, \max_{j=1} n\ min\ \{I_{a_{ij}}, I_{b_{jk}}\}, \min_{j=1} n\ max\ \{F_{a_{ij}}, F_{b_{jk}}\}\right)$.

3. SOLUTION PROCEDURE

In this section, we develop an approach based on max-min product and comparison matrix to deal with multi-criteria decision making problem with neutrosophic information. Let us consider this MCDM procedure adapted from Das S and Kar S [8].

Step 1: Neutrosophic choice matrix $\mu_n(i,j)$ and combined NCM $\mu_n^c(i,j)$ are computed for each of the decision makers d_n, $n = 1,2,\ldots,k$ based on their parameters.

Step 2: Max-min product of neutrosophic soft matrix M_A^n and combined choice matrix $\mu_n^c(i,j)$ are calculated for each decision makers.

Step 3: Find the aggregation of the product neutrosophic soft matrices $P_n \forall$ n.

Step 4: Formulate the comparison matrix C_A of the resultant neutrosophic soft matrix M_{NSM}.

Step 5: Obtain the score $\sigma_i$of NSM, the maximum score is the preferable choice of decision makers.

3.1 Numerical Example

Let us consider,$U = \{u_1, u_2, \ldots u_m\}$ be the set of alternatives, $E = \{e_1, e_2, \ldots e_p\}$ be the set of parameters, $D = \{d_1, d_2, \ldots d_n\}$ be the set of decision makers. Then the neutrosophic soft matrix M_A^n denotes the information provided by decision maker d_k , $k = 1,2,\ldots,n$. The assessment is represented in the form of neutrosophic number. Assume that, $M_A^n = (a_{ij})_{m\times n} = (T_{ij}, I_{ij}, F_{ij})$ is the decision matrix (a_{ij} is a netrosophic number) for alternative u_i, $i = 1,2,\ldots,m$ associated with the parameter e_j, $j = 1,2,\ldots,p$ given by decision maker d_k,$k = 1,2,\ldots,n$. The steps of neutrosophic multi-criteria decision making method can be presented as follows.

Let three investors $D = \{d_1, d_2, d_3\}$ jointly want to select a best investment portfolio. Let U be the universal set, $U = \{u_1, u_2, u_3\}$ be the set of alternatives, where,

u_1- Mutual funds

u_2- Bonds

u_3- Stocks

Let $\mathrm{E} = \{\mathrm{e}_1, \mathrm{e}_2, \mathrm{e}_3, \mathrm{e}_4, \mathrm{e}_5\}$ be the set of common criteria or parameters, where

e_1- Expected Return

e_2- Cost of investment

e_3- Market Conditions

e_4- Liquidity

e_5- Risk Tolerance

Among three experts, d_1 interested to evaluate the parameters (e_2, e_3, e_4). d_2 interested to evaluate the parameters (e_1, e_2, e_5) and d_3 evaluate the parameters (e_1, e_3, e_4, e_5). Opinions of three experts represented in three different neutrosophic soft matrices given below,

$$M_A^1 = \begin{bmatrix} (0,0,0) & (0.2,0.5,0.7) & (0.8,0.7,0.6) & (0.3,0.6,0.7) & (0,0,0) \\ (0,0,0) & (0.4,0.7,0.8) & (0.3,0.6,0.1) & (0.7,0.2,0.3) & (0,0,0) \\ (0,0,0) & (0.6,0.8,0.2) & (0.4,0.2,0.8) & (0.8,0.1,0.4) & (0,0,0) \end{bmatrix}$$

$$M_A^2 = \begin{bmatrix} (0.5,0.7,0.8) & (0.2,0.5,0.7) & (0,0,0) & (0,0,0) & (0.6,0.4,0.1) \\ (0.3,0.5,0.1) & (0.4,0.7,0.8) & (0,0,0) & (0,0,0) & (0.7,0.8,0.1) \\ (0.6,0.8,0.1) & (0.6,0.8,0.2) & (0,0,0) & (0,0,0) & (0.8,0.5,0.3) \end{bmatrix}$$

$$M_A^3 = \begin{bmatrix} (0.5,0.7,0.8) & (0,0,0) & (0.8,0.7,0.6) & (0.3,0.6,0.7) & (0.6,0.4,0.1) \\ (0.3,0.5,0.1) & (0,0,0) & (0.3,0.6,0.1) & (0.7,0.2,0.3) & (0.7,0.8,0.1) \\ (0.6,0.8,0.1) & (0,0,0) & (0.4,0.2,0.8) & (0.8,0.1,0.4) & (0.8,0.5,0.3) \end{bmatrix}$$

Step 1: From the definition 2.4, neutrosophic choice matrices given below,

$$\mu_1(i,j) = \begin{bmatrix} (0,0.5,1) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1) \\ (0,0.5,1) & (1,0.5,0) & (1,0.5,0) & (1,0.5,0) & (0,0.5,1) \\ (0,0.5,1) & (1,0.5,0) & (1,0.5,0) & (1,0.5,0) & (0,0.5,1) \\ (0,0.5,1) & (1,0.5,0) & (1,0.5,0) & (1,0.5,0) & (0,0.5,1) \\ (0,0.5,1) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1) \end{bmatrix}$$

$$\mu_2(i,j) = \begin{bmatrix} (1,0.5,0) & (1,0.5,0) & (0,0.5,1) & (0,0.5,1) & (1,0.5,0) \\ (1,0.5,0) & (1,0.5,0) & (0,0.5,1) & (0,0.5,1) & (1,0.5,0) \\ (0,0.5,1) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1) \\ (0,0.5,1) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1) \\ (1,0.5,0) & (1,0.5,0) & (0,0.5,1) & (0,0.5,1) & (1,0.5,0) \end{bmatrix}$$

$$\mu_3(i,j) = \begin{bmatrix} (1,0.5,0) & (0,0.5,1) & (1,0.5,0) & (1,0.5,0) & (1,0.5,0) \\ (0,0.5,1) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1) \\ (1,0.5,0) & (0,0.5,1) & (1,0.5,0) & (1,0.5,0) & (1,0.5,0) \\ (1,0.5,0) & (0,0.5,1) & (1,0.5,0) & (1,0.5,0) & (1,0.5,0) \\ (1,0.5,0) & (0,0.5,1) & (1,0.5,0) & (1,0.5,0) & (1,0.5,0) \end{bmatrix}$$

Using Definition 2.4, combined choice matrices are given below,

$$\mu_1^c(i,j)=\begin{bmatrix}(0,0.5,1) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1)\\ (1,0.5,0) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1) & (1,0.5,0)\\ (1,0.5,0) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1) & (1,0.5,0)\\ (1,0.5,0) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1) & (1,0.5,0)\\ (0,0.5,1) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1)\end{bmatrix}$$

$$\mu_2^c(i,j)=\begin{bmatrix}(0,0.5,1) & (0,0.5,1) & (1,0.5,0) & (1,0.5,0) & (0,0.5,1)\\ (0,0.5,1) & (0,0.5,1) & (1,0.5,0) & (1,0.5,0) & (0,0.5,1)\\ (0,0.5,1) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1)\\ (0,0.5,1) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1)\\ (0,0.5,1) & (0,0.5,1) & (1,0.5,0) & (1,0.5,0) & (0,0.5,1)\end{bmatrix}$$

$$\mu_3^c(i,j)=\begin{bmatrix}(0,0.5,1) & (1,0.5,0) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1)\\ (0,0.5,1) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1)\\ (0,0.5,1) & (1,0.5,0) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1)\\ (0,0.5,1) & (1,0.5,0) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1)\\ (0,0.5,1) & (1,0.5,0) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1)\end{bmatrix}$$

Step 2: Now calculate the product of NSM M_A^n and combined choice matrix $\mu_n^c(i,j)$ for each decision makers $d_n, n=1,2,3$ are,

$$P_1 = M_A^1 \otimes \mu_1^c(i,j)$$

$$=\begin{bmatrix}(0.8,0.5,0.6) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1) & (0.8,0.5,0.6)\\ (0.7,0.5,0.1) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1) & (0.7,0.5,0.1)\\ (0.8,0.5,0.2) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1) & (0.8,0.5,0.2)\end{bmatrix}$$

$$P_2 = M_A^2 \otimes \mu_2^c(i,j)$$

$$=\begin{bmatrix}(0,0.5,1) & (0,0.5,1) & (0.6,0.5,0.1) & (0.6,0.5,0.1) & (0,0.5,1)\\ (0,0.5,1) & (0,0.5,1) & (0.7,0.5,0.1) & (0.7,0.5,0.1) & (0,0.5,1)\\ (0,0.5,1) & (0,0.5,1) & (0.8,0.5,0.1) & (0.8,0.5,0.1) & (0,0.5,1)\end{bmatrix}$$

$$P_3 = M_A^3 \otimes \mu_1^c(i,j)$$

$$=\begin{bmatrix}(0,0.5,1) & (0.8,0.5,0.1) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1)\\ (0,0.5,1) & (0.7,0.5,0.1) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1)\\ (0,0.5,1) & (0.8,0.5,0.1) & (0,0.5,1) & (0,0.5,1) & (0,0.5,1)\end{bmatrix}$$

Step 3: Aggregation of the product NSMs given below,

$$M_{NSM} = P_1 \oplus P_2 \oplus P_3$$

$$=\begin{bmatrix}(0.8,0.5,0.6) & (0.8,0.5,0.1) & (0.6,0.5,0.1) & (0.6,0.5,0.1) & (0.8,0.5,0.6)\\ (0.7,0.5,0.1) & (0.7,0.5,0.1) & (0.7,0.5,0.1) & (0.7,0.5,0.1) & (0.7,0.5,0.1)\\ (0.8,0.5,0.2) & (0.8,0.5,0.1) & (0.8,0.5,0.1) & (0.8,0.5,0.1) & (0.8,0.5,0.2)\end{bmatrix}$$

Step 4: Now formulate the comparison matrix,

$$C_A = \begin{matrix} & e_1 & e_2 & e_3 & e_4 & e_5 \\ u_1 & 2 & 2 & 0 & 0 & 2 \\ u_2 & 2 & 0 & 1 & 1 & 2 \\ u_3 & 3 & 2 & 2 & 2 & 3 \end{matrix}$$

Step 5: Using score value, $\sigma_i = \sum j\ c_{ij}$ on the above matrix we get,

Table 1: Score value of each alternative

Score	σ_i
u_1	6
u_2	6
u_3	12

The maximum score value is 12 (u_3), Clearly, the alternative u_3 is selected as the collective decision of all the three decision makers.

Interpretation and Suggestion

According to the results above, Stocks (u_3) are the best option of all other altenative. Investments with less risk are best made in stocks. For investors who can afford less risk and expect more return , stocks are the ideal investment. This solving method helps the decision makers to take better decision for their combined opinions. This approach provides precise solutions to MCDM problems and applicable to problems arising in real life situations.

4.CONCLUSION

In this study, a new multi-criteria decision making method based on neutrosophic soft matrix is proposed and it is applied to develop a model and to evaluate the investment portfolio selection. Here, the concept of comparison matrix and score value are used to obtain the optimum solution. This MCDM procedure will be applied to further extensions of neutrosophic soft sets in future.

REFERENCES

1. L.A. Zadeh, Fuzzy set, *Information and control*.vol 8 (1965), 338-353.
2. K.Attenssov, Intuitionistic fuzzy set, *Fuzzy sets and systems*.Vol 20 (1986).87-96.
3. F.Smarandache, Neutrosophy, Neutrosophic probability, set and logic, *Amer.Res.press,Rehoboth*, USA (1998).
4. D.Molodtsove, Soft Set Theory-First results,*Computers and Mathematics with Applications* 37,(4-5):19-31, (1999).
5. P.K.Maji, Neutrosophic Soft set, *Annals of Fuzzy Mathematics and Informatics*, Vol 5, (2013), 157-168.

6. F.Smarandache, Neutrosophic set, A generalization of the Intuitionistic fuzzy set, *International Journal of Pure and Applied Mathematics*. 24(3), (2005).

7. Sujit Das,Samarjit Kar, Group decision making in medical system, an Intuitionistic fuzzy soft set approach. *Appl.Soft comput* 24,(2014).

8. Das S.,Kar S., Group decision making using neutrosophic soft matrix: An algorithmic approach. *Appl. Soft comput and information sciences*. 9, (2017).

9. Murat Kirisa, Necip Simsek, Neutrosophic soft sets with medical decision making applications. *Sigma J Eng and Nat Sci*,10, (2019).